Exercice 1

- $u_1(x,t) = \cos(kat)\sin(kx)$

$$\frac{\partial u_1}{\partial t} = -ka\sin(kat)\sin(kx) \qquad \frac{\partial^2 u_1}{\partial t^2} = -k^2a^2\cos(kat)\sin(kx)$$

$$\frac{\partial u_1}{\partial x} = k\cos(kat)\cos(kx) \qquad \frac{\partial^2 u_1}{\partial x^2} = -k^2\cos(kat)\sin(kx)$$

$$\Longrightarrow \frac{\partial^2 u_1}{\partial t^2} = a^2\frac{\partial^2 u_1}{\partial x^2}. \qquad \begin{cases} u_1(x,0) = \sin(kx) \\ \frac{\partial u_1}{\partial t}(x,0) = 0 \end{cases}$$

- $u_2(x,t) = \sin(kat)\sin(kx)$

$$\frac{\partial u_2}{\partial t} = ka\cos(kat)\sin(kx) \qquad \frac{\partial^2 u_2}{\partial x^2} = -ka^2\sin(kat)\sin(kx)$$

$$\frac{\partial u_2}{\partial x} = k\sin(kat)\cos(kx) \qquad \frac{\partial^2 u_2}{\partial x^2} = -k^2\sin(kat)\sin(kx)$$

$$\Longrightarrow \frac{\partial^2 u_2}{\partial t^2} = a_2\frac{\partial^2 u_2}{\partial x^2} \qquad \begin{cases} u_2(x,0) = 0 \\ \frac{\partial u_2}{\partial t}(x,0) = ka\sin(kx) \end{cases}$$

Exercice 2

- Par la formule de D'Alembert,

$$u(t,x) = \frac{1}{2}(\cos(kx+kat) + \cos(kx-kat))$$

$$= \frac{1}{2}(\cos(kx)\cos(kat) - \sin(kx)\sin(kat) + \cos(kx)\cos(kat) + \sin(kx)\sin(kat))$$

$$= \cos(kx)\cos(kat).$$

- Encore par la formule,

$$u(t,x) = \frac{1}{2a}\int_{x-at}^{x+at}\cos(ks)\,ds = \frac{1}{2ak}\Big[\sin(ks)\Big]_{x-at}^{x+at}$$

$$= \frac{1}{2ak}(\sin(kx+kat) - \sin(kx-kat))$$

$$= \frac{1}{2ak}\begin{pmatrix}\sin(kx)\cos(kat) + \cos(kx)\sin(kat) \\ -\sin(kx)\cos(kat) + \cos(kx)\sin(kat)\end{pmatrix} = \frac{1}{ak}\cos(kx)\sin(kat).$$

Exercice 3

$$u_1(x,t) = A_1 \cos(kx \pm kat)$$

$$= \underbrace{A_1 \cos(kx)\cos(kat)}_{2.1} \mp \underbrace{A_1 \sin(kx)\sin(kat)}_{1.2}$$

$$u_2(x,t) = A_2 \sin(kx \pm kat)$$

$$= \underbrace{A_2 \sin(kx)\cos(kat)}_{1.1} \pm \underbrace{A_2 \cos(kx)\sin(kat)}_{2.2}$$