EXERCISE SHEET 9

WRITTEN SOLUTIONS OF EXERCISES 1.2 AND 1.15 TO BE PRESENTED ON 4/12

Exercise 1. For each of the following functions f, determine the intervals on which f is monotone increasing, monotone decreasing, the local minima and the local maxima of f. Sketch a picture of the graph of f. (*Hint: do not forget to consider the domain of definition and the limits of* f) .

(1) $f(x) = 6x + 2$.
(2) $f(x) = -x^2 + x$.
(3) $f(x) = 2x^2 + x - 6$.
(4) $f(x) = 4x^3 + 3x^2 - 6x + 2$.
(5) $f(x) = 1 - 12x - 9x^2 - 2x^3$.
(6) $f(x) = 3 - 2x^2 + 4x^4$.
(7) $f(x) = \dfrac{2x}{x^2+1}$.
(8) $f(x) = \sinh(x)$.
(9) $f(x) = \cosh(x)$.
(10) $f(x) = \dfrac{1}{\cosh(x)}$.
(11) $f(x) = \tanh(x)$.
(12) $f(x) = \dfrac{x-1}{x^2}$.
(13) $f(x) = \dfrac{x^2}{2-x}$.
(14) $f(x) = \dfrac{3x+2}{1-x^2}$.
(15) $f(x) = \dfrac{x^3}{x+1}$.
(16) $f(x) = \sqrt{1-x}$.
(17) $f(x) = x + e^{-x}$.
(18) $f(x) = \ln(1-x)$.
(19) $f(x) = \ln(x^2-4)$.
(20) $f(x) = x\ln(x)$.

Exercise 2. Let $f : \mathbb{R} \to \mathbb{R}$ be any function, and let $g(x) = e^{f(x)}$.

(1) Supposing that f is differentiable, show that for every $x \in \mathbb{R}$, $f'(x) > 0$ if and only if $g'(x) > 0$.
(2) Show that f and g have the same local minima and maxima.
(3) Now show, applying the definition of monotonicity, that f is monotone increasing if and only if g is monotone increasing. (*Warning: we are not supposing that f is differentiable.*)
(4) Finally, show that for every f and g monotone increasing, $f \circ g$ is monotone increasing.

Exercise 3. Show that, if $f : I \to \mathbb{R}$ is a monotone decreasing function, then $f' \leq 0$. Find a function f which is strictly decreasing but f' is not strictly negative.