EXERCISE SHEET 12

Exercise 1. Use Taylor approximations to compute the following limits:

(1) $\lim_{x\to 0} \dfrac{e^x - 1}{x}$

(2) $\lim_{x\to 0} \dfrac{e^x - 1 - x}{x^2}$

(3) $\lim_{x\to 0} \dfrac{\sinh(x)}{x}$

(4) $\lim_{x\to 0} \dfrac{\cosh(x) - 1}{x^2}$

(5) $\lim_{x\to 0} \dfrac{\tan(x) - x}{x^3}$

(6) $\lim_{x\to 0} \dfrac{x - \arctan(x)}{x^3}$

(7) $\lim_{x\to 0} \dfrac{\sin(x) - \arctan(x)}{x^3}$

(8) $\lim_{x\to 0} \dfrac{\ln(1 + x^3)}{x^3}$

(9) $\lim_{x\to 0} \dfrac{(2 - 2\cos(x))^3}{x^6}$

Exercise 2. Compute the first three values obtained by Newton's method to find a zero of the following functions, with initial point x_0. Compare the obtained value with a solution that you may find by another method.

(1) $f(x) = x^2 - 2 \qquad x_0 = 2$;

(2) $f(x) = \cos(x) \qquad x_0 = 2$;

(3) $f(x) = x^2 + x - 1 \quad x_0 = 0$;

(4) $f(x) = x^2 + x - 1 \quad x_0 = 2$;

(5) $f(x) = \tan(x) - 1 \quad x_0 = 0$;

(6) $f(x) = \tan(x) - 1 \quad x_0 = 1$.