EXERCISE SHEET 1

Exercise 1. Find (if it exists) the limit

$$\ell = \lim_{n\to+\infty} a_n$$

of the following sequences a_n. If the limit exists, prove that the limit is the value given in your solution by applying the definition of limit. If the limit does not exists, prove that indeed a_n has no limit.

(1) $a_n = 2$
(2) $a_n = 1/n^2$
(3) $a_n = 1 - \frac{1}{n}$
(4) $a_n = n^3$
(5) $a_n = \cos(n\pi)$
(6) $a_n = n!$ (*More difficult. Check the definition of factorial on wikipedia if necessary.*)

Exercise 2. For each of the sequences a_n in the previous exercise, define the new sequences b_n and c_n as follows:

$$b_n = \begin{cases} 2\pi/\sqrt{3} & \text{if } n = 173560687 \\ a_n & \text{if } n \neq 173560687 \end{cases} \qquad c_n = \begin{cases} a_n & \text{if } n \leq 6453867024576 \\ 0 & \text{if } n > 6453867024576 \end{cases}.$$

For every point of the previous exercise, find the limit of the new sequences b_n and c_n.

Exercise 3. Find the supremum and infimum of the following sets.

(1) $\mathbb{N}$
(2) $\mathbb{Z}$
(3) $\{x \in \mathbb{R} : x < 4\}$
(4) $\{x \in \mathbb{Q} : |x| < 34\}$
(5) $\{x \in \mathbb{R} : x^2 < 4\}$
(6) $\{x \in \mathbb{Q} : x^2 < 34\}$

Exercise 4. Give the definition of monotone **decreasing** sequence. Then prove the following theorem: any monotone decreasing sequence a_n admits a limit

$$\ell = \lim_{n\to+\infty} a_n$$

(which is possibly $-\infty$). *Hint: consider the new sequence $b_n = -a_n$ and apply the monotone convergence theorem for increasing sequences.*