Extra: Forme locale canonique

Soit $\gamma : I \to \mathbb{R}$ paramétrée par longueur d'arc. Supposons $t_0 = 0$.

Par Taylor,

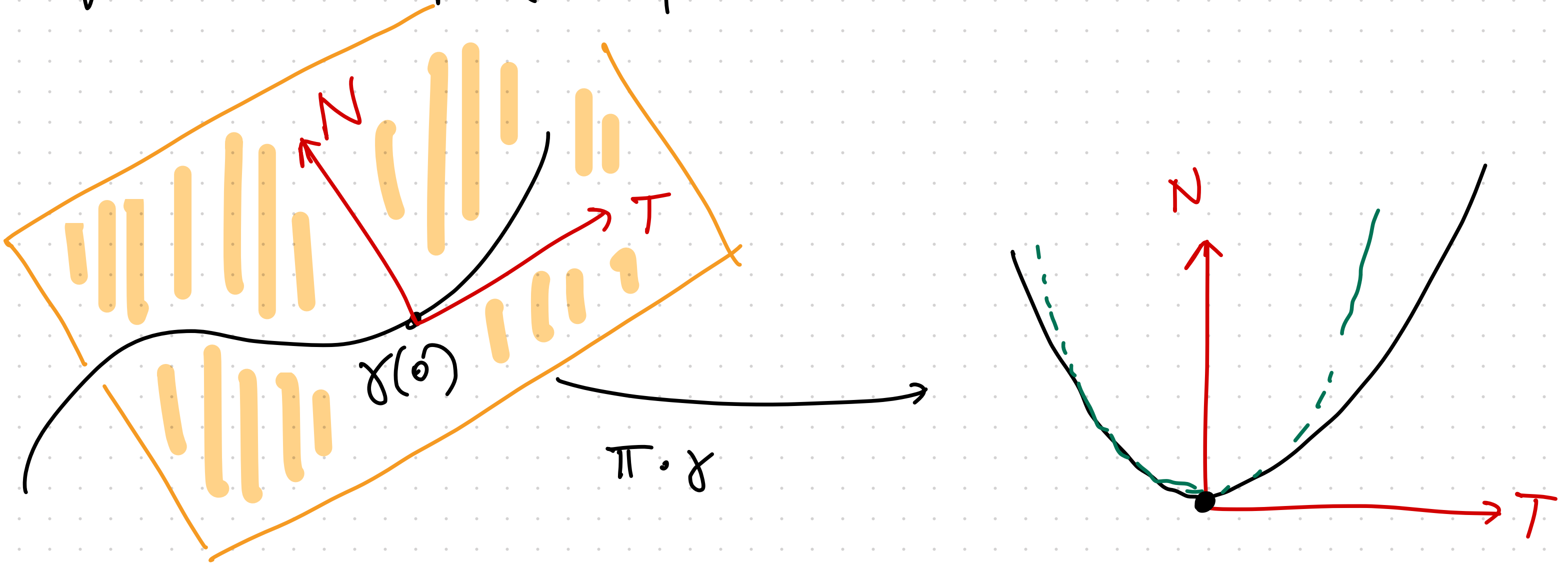
$$\gamma(t) = \gamma(0) + \gamma'(0)t + \frac{\gamma''(0)}{2}t^2 + \frac{\gamma'''(0)}{6}t^3 + O(t^4)$$

$$= \gamma(0) + T(0)t + \underbrace{T'(0)}_{= k(0)N(0)}\frac{t^2}{2} + \underbrace{T''(0)}_{\substack{= k'(0)N(0) + k(0)N'(0) \\ = k'(0)N(0) + k(0)\left(-k(0)T(0) + \tau(0)B(0)\right)}}\frac{t^3}{6} + O(t^4)$$

Donc :

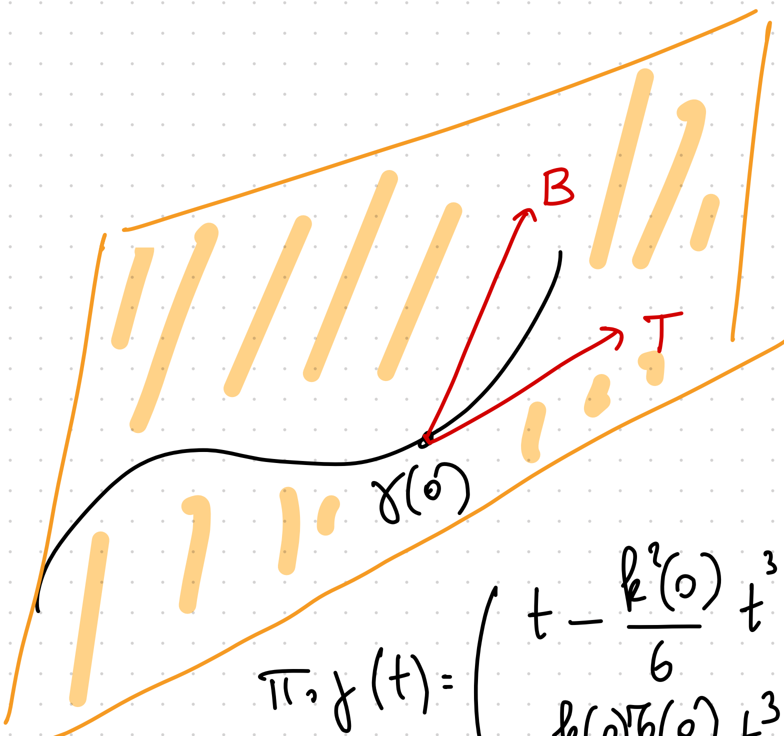
$$\gamma(t) = \gamma(0) + T(0)\left(t - \frac{k^2(0)}{6}t^3\right) + N(0)\left(\frac{k(0)}{2}t^2 + \frac{k'(0)}{6}t^3\right) + B(0)\left(\frac{k(0)\tau(0)}{6}t^3\right) + O(t^4)$$

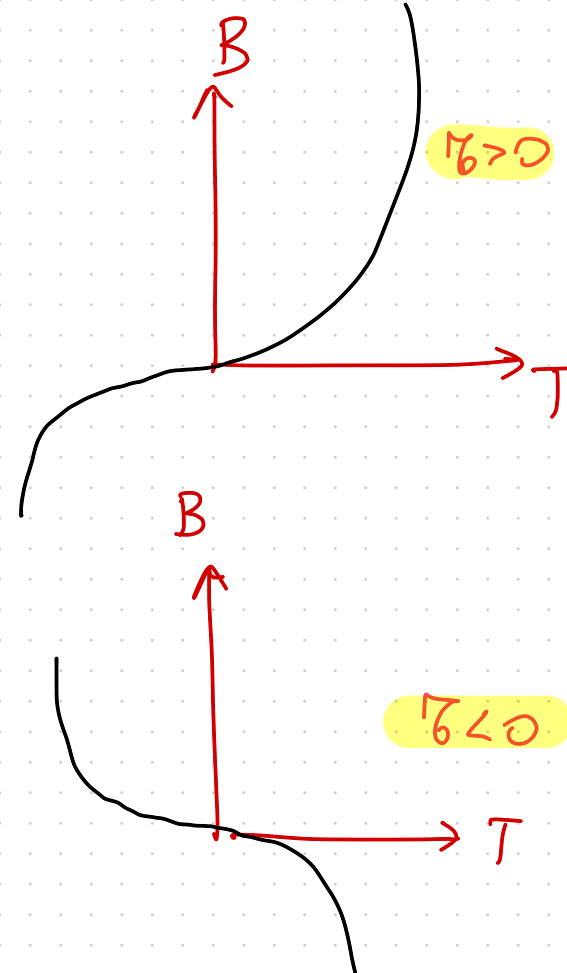
• Projection sur $\mathrm{Span}(T(0), N(0))$



$$\pi \circ \gamma(t) = \underbrace{\begin{pmatrix} t \\ \frac{k(0)}{2} t^2 \end{pmatrix}}_{\text{parabole } y = \frac{k(0)}{2} x^2} + O(t^3)$$

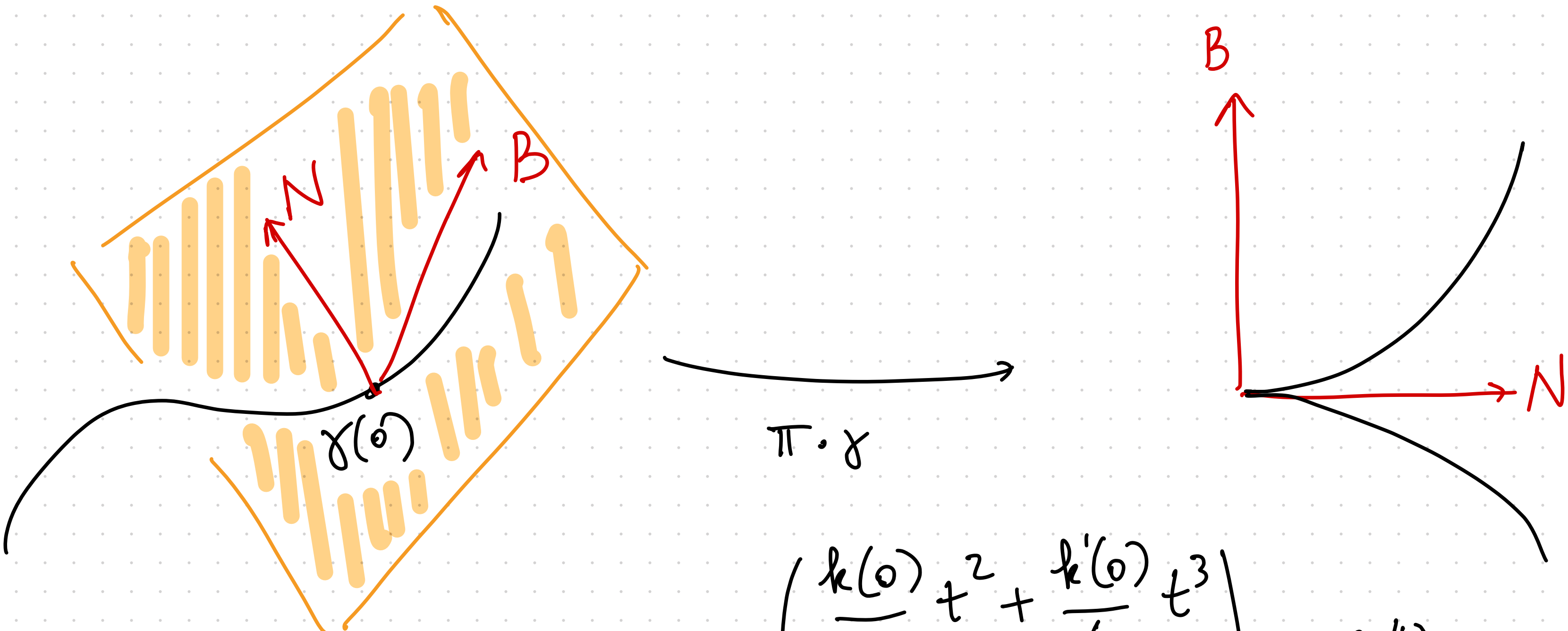
- Projection sur $\mathrm{Span}(T(0), B(0))$





$$\pi \circ \gamma(t) = \begin{pmatrix} t - \dfrac{k^2(0)}{6} t^3 \\ \dfrac{k(0)\tau(0)}{6} t^3 \end{pmatrix} + O(t^4)$$

• Projection sur $\mathrm{Span}(N(0), B(0))$



$$\pi \circ \gamma(t) = \begin{pmatrix} \frac{k(0)}{2} t^2 + \frac{k'(0)}{6} t^3 \\ \frac{k(0)\tau(0)}{6} t^3 \end{pmatrix} + O(t^4)$$