PROBLEM SHEET ON LINEAR ALGEBRA AND DYNAMICAL SYSTEMS

Problem 1. *Consider the matrices*

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 2 \\ 0 & 0 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & -1 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}.$$

For each matrix A, B, C, find the eigenvalues and the corresponding eigenspaces, and decide if the matrix is diagonalizable.

Problem 2. *Consider the following matrix, depending on a parameter $h \in \mathbb{R}$:*

$$A(h) = \begin{pmatrix} 2+h & 1 & 1 \\ 1 & h & 1 \\ h & 1 & h \end{pmatrix}.$$

- *For which values of h is the matrix $A(h)$ invertible?*
- *For which values of h is the matrix $A(h)$ symmetric?*
- *Discuss the value of the rank of $A(h)$, as h varies in $\mathbb{R}$.*
- *For which values of h is the vector $(1, 1, 1)$ in the range of $A(h)$?*

Problem 3. *Using the method of the eigenvalues, find the solution of the linear system of ODE given by*

$$\begin{cases} x'(t) = -2x(t) + 2y(t) \\ y'(t) = 2x(t) - 5y(t) \end{cases} \qquad (\text{福气})$$

with the initial value

$$\begin{cases} x'(0) = 6 \\ y'(0) = 13 \end{cases}.$$

Find the critical points of (福气) *and determine their type.*

Problem 4. *Using the method of the eigenvalues, find the general solution of the following linear system of ODE:*

$$\begin{cases} x'(t) = 3x(t) \\ y'(t) = -2z(t) \\ z'(t) = 2y(t) \end{cases}.$$

Extra challenge: draw a three-dimensional picture of the orbits of the system.

Date: September 16, 2015.