TEST - SEPTEMBER 15th, 2015

SURNAME:

NAME:

Instruction: Please read carefully the following instructions:

- remember to write your name;
- you have 3 hours to complete the test;
- neither calculators nor notes nor books are allowed;
- if you do not understand some requests, please raise you hand;
- fill in the blanks with your answer and return both this sheet and the detailed solutions of the problems.

PROBLEMS

Problem 1. Find the solution, the critical points and their type for the following initial value problem:

$$\begin{cases} x' = -4y \\ y' = x \end{cases} \qquad \text{with} \qquad \left(\begin{array}{c} x(0) \\ y(0) \end{array} \right) = \left(\begin{array}{c} 0 \\ 1 \end{array} \right)$$

$x(t) =$

$y(t) =$

critical points and their type:

Problem 2. Depending on $0 \leqslant a \in \mathbb{R}$, find the solution and its interval of existence of the following initial value problem:

$$\begin{cases} u' = \dfrac{1 - t}{u} \\ u(0) = \sqrt{a} \end{cases}$$

$u(t) =$

$I =$

Problem 3. Find the solution and its interval of existence of the following initial value problem:

$$\begin{cases} u' = u\sin t + \sin(2t) \\ u(0) = -2 \end{cases}$$

$u(t) =$

$I =$

Problem 4. Depending on $0 \leqslant a \in \mathbb{R}$, find the general solution and its interval of existence of the following equation:

$$u^{(3)} + (2a - 4)u'' + a^2 u' = 0$$

The solution is...

Problem 5. Find the general solution and its interval of existence of the following equation:

$$y'' + 4y = \cos(2t)$$

$y(t) =$

$I =$

Problem 6. Find for which value of the parameter $k \in \mathbb{R}$
- the matrix $A = \begin{pmatrix} k & k - \frac{2}{3} \\ 3 & k \end{pmatrix}$ is invertible;
- the vector $v = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is in the range of A;
- the vector $w = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ is in the kernel of A.

A is invertible for...

v is in the range of A for...

w is in the kernel of A for...